High Accuracy Formulas for Calculation of the Characteristic Impendance of Microstrip Lines

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***Abstract*— An analytical formula for determination of the characteristic impedance of a microstrip line assuming the quasi-TEM mode of propagation is presented. The new form of the final formulas contains only integrals which can be numerically performed by means of the Gauss–Laguerre quadrature. The method can be applied to multilayer lines and also to the case of anistropic dielectrics. By using some suitable conformal mappings the formulas obtained can be used to determine the characteristic impedance of some cylindrical microstrip lines. We have compared the results given by the proposed formulas with the finite analytical solution available in a particular case and also with results obtained by the substrip method. All the performed tests indicate that the proposed formulas are highly accurate and efficient relations for determining the characteristic impedance of microstrip lines.**

I. Introduction

There is a vast amount of literature on the numerical computation of the characteristic impedance of microstrip. Wheeler [1] used an approximate conformal mapping method to calculate the capacitance of a mixed dielectric media microstrip. Sylvester [2], Bryant and Weiss [3], and Farrar [4] treated the problem by the method of moments and dielectric Green's function. Yamashita and Mittra [5] presented an analysis based on a variational principle. Analysis of various planar transmission lines have been carried out in spectral domain by Itoh and Mittra [6] and Itoh [7]. Poh *et al.* [8] considered the solution for the line capacitance of a microstrip by means of a spectral domain analysis method.

J. F. Fikioris *et al.* [9] have given an exact solution for the shielded printed microstrip lines by the Carleman–Vekua method. Cheng and Everard [10] proposed a new method based on the spectral domain approach. Medina and Horno in [11] proposed two different approaches to speed up the evaluation of spectral series. We mention also the paper [12] where an analytic method was given for determining the capacitance matrix of multiconductor planar and cylindrical lines.

In [13] an analytical method for solving the covered microstrip problem was given. The solution is exact but it is expressed by means of the solution of an infinite system of linear equations. The numerical examples provide a very good approximation even in the case we consider only the first two equations in the infinite set of linear equations.

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Auda in [14] introduced a new cylindrical microstrip line. It consists of an infinitesimally thin strip on the surface of a dielectric cylinder partially embedded in a perfectly conducting ground plane.

In this paper we give a new form of the formulas for computing the capacitance of a covered microstrip line. The new form is simpler than the older one and also considerably increases the precision. Further on, it is shown how the method applies to the microstrips of multilayered substrate and to the case of anisotropic dielectrics. By some suitable conformal mappings the method can also be applied to the cylindrical microstrip line considered in [14]. Also, there are given some other examples of cylindrical microstrip lines which can be analyzed by considering the equivalent planar lines resulting from some conformal mappings.

To show how the method works, we applied it to some planar and cylindrical structures. The first example consists of a symmetrical covered microstripline. In this case a finite analytical formula in terms of elliptical functions is available. The comparison with exact solution shows that the error is less than 0.07% through the range of microstrips of practical interest. As a second application we computed the characteristic impedance of an open microstrip. The obtained results have shown that it is possible to compute the value Z_0 with remarkable precision by using the proposed formulas. There are given also some graphs for the physical characteristics of the cylindrical microstrip line considered by Auda [14].

II. Computational Formulas

We consider microstrip problems which can be converted to the system of integral equations

$$\int_0^\infty B(k)K(k)\cos(kx)dk = V_0,\ x \in (-b,\, b) \qquad (1)$$

$$\int_0^\infty B(k)\sin(kx)dk = \frac{q_0}{2(\epsilon_1+\epsilon_2)}\,\mathrm{sgn}(x),\quad x \in (-\infty,\, -b)\cup(b,\, \infty) \qquad (2)$$

where V_0 and q_0 are the strip potential and charge, respectively. The locally integrable function $K(k)$ is assumed to be of the form

$$K(k) = 1 - \tilde{\eta}(k)\exp(-2kh) \qquad (3)$$

where $h \geq 0$ and

$$\lim_{k\to\infty} \tilde{\eta}(k) = \tilde{\eta}_0. \qquad (4)$$

The analysis carried out in [13] shows that the capacity of the microstrip is given by the relation

$$C = \frac{2(\epsilon_1 + \epsilon_2)}{\frac{2}{\pi}\ln(2) - \tilde{b}_0} \tag{5}$$

where $\tilde{b}_0$ results by solving an infinite system of linear equations. A good approximation can be obtained by considering only the first two equations in the infinite linear system

$$\tilde{b}_0 = \tilde{a}_0 + 2\pi \frac{\tilde{a}_1^2}{1 - \tilde{a}_{11}}. \tag{6}$$

The coefficients entering in this formula can be computed by relations

$$\tilde{a}_0 = \frac{2}{\pi}\int_0^\infty \frac{J_0^2(kb)\tilde{\eta}(k)\exp(-2kh) - \exp(-kb)}{k}\,dk \tag{7}$$

$$\tilde{a}_1 = \frac{2}{\pi}\int_0^\infty \frac{J_0(kb)J_2(kb)}{k}\tilde{\eta}(k)\exp(-2kh)\,dk \tag{8}$$

$$\tilde{a}_{11} = 4\int_0^\infty \frac{J_2^2(kb)}{k}\tilde{\eta}(k)\exp(-2kh)\,dk. \tag{9}$$

Here J_0, J_1, J_2 are the corresponding Bessel functions of the first kind. The formulas (5), (6), (8), and (9) coincide with some relations given in [13]. The coefficient $\tilde{a}_0$ in this new form is more suitable for numeric computation. A method for computing the integrals (7)–(9) shall be given in Section VIII.

In the case of the quasi-TEM mode of propagation and where the line has negligible loss, the characteristic impedance Z_0 of the microstrip line is given by

$$Z_0 = \frac{1}{v\sqrt{CC_0}} \tag{10}$$

where v is the velocity of light in vacuum, C is the capacitance per unit length of the given microstrip, and C_0 is the capacitance per unit length for the same structure but with $\epsilon_1 = \epsilon_2 = \epsilon_0$.

III. Classical Microstrip Problem

In the case of the covered microstrip line consisting of a conducting strip of zero thickness placed on a dielectric substrate (Fig. 1), the kernel $K(k)$ in (1) is [13]

$$K(k) = \frac{\epsilon_1 + \epsilon_2}{\epsilon_1\coth(kh_1) + \epsilon_2\coth(kh_2)}. \tag{11}$$

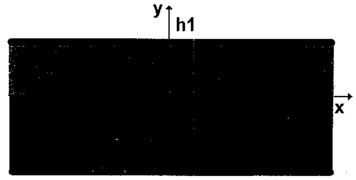

Fig. 1. Covered microstrip line.

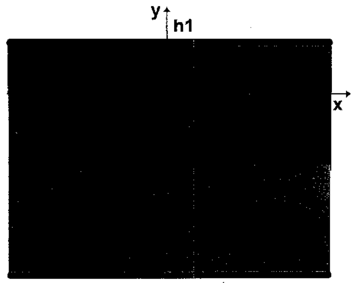

Fig. 2. Multilayered microstrip line.

Hence there is (12) as shown at the bottom of the page where

$$h = \min(h_1, h_2),\ \epsilon_r = \epsilon_2/\epsilon_1.$$

IV. Microstrip on Multilayered Substrate

To show how the method applies in the case of multilayered microstrips we consider the microstrip line with two dielectric layers shown in Fig. 2. In this case the expressions for the potentials in domains D_1, D_2, D_3 are

$$V^{(1)}(x, y) = \int_0^\infty A(k)\frac{\sinh(k(h_1 - y))}{\sinh(kh_1)}\cos(kx)\,dk \tag{13}$$

and (14) and (15) as shown at the bottom of the page.

These potentials satisfy the continuity condition along the surfaces $y = 0$ and $y = -h_2$, the boundary conditions along the planes $y = h_1$ and $y = -(h_2 + h_3)$ and the charge free condition along the plane $y = -h_2$. By imposing the obvious conditions along the circuit interface $y = 0$ we get

$$K(k) = \frac{\epsilon_1 + \epsilon_2}{\epsilon_1\coth(kh_1) + \epsilon_2\Delta_2} \equiv \frac{\epsilon_1 + \epsilon_2}{\Delta} \tag{16}$$

and hence

$$\tilde{\eta}(k) = 2\left[\epsilon_1\frac{\exp(2k(h - h_1))}{(1 - \exp(-2kh_1))} + \epsilon_2\frac{\exp(2k(h - h_2))}{(1 - \exp(-2kh_2))}\Delta_1\right]/\Delta \tag{17}$$

$$\tilde{\eta}(k) = \frac{2\exp(2k(h - h_1))/(1 - \exp(-2kh_1)) + 2\epsilon_r\exp(2k(h - h_2))/(1 - \exp(-2kh_2))}{\coth(kh_1) + \epsilon_r\coth(kh_2)} \tag{12}$$

$$V^{(2)}(x, y) = \int_0^\infty A(k)\left\{\frac{\epsilon_2 + \epsilon_3\coth(kh_2)\coth(kh_3)}{\epsilon_2\coth(kh_2) + \epsilon_3\coth(kh_3)}\sinh(ky) + \cosh(ky)\right\}\cos(kx)\,dk \tag{14}$$

$$V^{(3)}(x, y) = \int_0^\infty A(k)\frac{\epsilon_2\cosh(2kh_2)\sinh(k(h_2 + h_3 + y))}{\epsilon_2\sinh(kh_3)\cosh(kh_2) + \epsilon_3\sinh(kh_2)\cosh(kh_3)}\cos(kx)\,dk \tag{15}$$

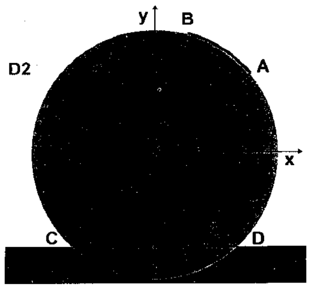

Fig. 3. Cylindrical microstrip line partially embedded in a perfectly conducting plane.

where again we have denoted $h = \min(h_1, h_2)$ and also

$$\Delta_1 = \frac{\epsilon_3 \coth(kh_3) - \epsilon_2}{\epsilon_2 \coth(kh_2) + \epsilon_3 \coth(kh_3)}, \quad \Delta_2 = \frac{\epsilon_2 + \epsilon_3 \coth(kh_2)\coth(kh_3)}{\epsilon_2 \coth(kh_2) + \epsilon_3 \coth(kh_3)}.$$

V. The Microstrip on Anisotropic Dielectric

In the case the domain D_2 in Fig. 1 is filled with an unisotropic dielectric characterized by permittivity tensor

$$\hat{\epsilon}_2 = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & 0 \\ \epsilon_{21} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \tag{18}$$

we can write

$$V^{(1)}(x, y) = \int_0^\infty A(k) \frac{\sinh(k(h_1 - y))}{\sinh(kh_1)} \cos(kx)\, dk \tag{19}$$

$$V^{(2)}(x, y) = \int_0^\infty A(k) \frac{\sinh(k(h_e + K_2 y))}{\sinh(kh_e)} \cdot \cos\left(kx - \frac{\epsilon_{22}}{\epsilon_{11}} y\right) dk \tag{20}$$

where

$$\epsilon_e = \sqrt{\epsilon_{11}\epsilon_{22} - \epsilon_{12}^2} \tag{21}$$

$$h_e = h_2 K_2, \; K_2 = \frac{\epsilon_e}{\epsilon_{22}}. \tag{22}$$

By imposing the boundary condition on the circuit plane $y = 0$ we get the expression

$$K(k) = \frac{\epsilon_1 + \epsilon_e}{\epsilon_1 \coth(kh_1) + \epsilon_e \coth(kh_e)} \tag{23}$$

and also in (24) shown at the bottom of the page.

We have now $h = \min(h_1, h_e)$.

VI. Cylindrical Microstrip Line Partially Embedded in a Ground Plane

The cylindrical line we consider in this section consists of an infinitesimally thin strip on the surface of a dielectric cylinder partially embedded in a perfectly conducting ground plane (see Fig. 3). In the particular case $\gamma = 0$ this problem was considered by Auda [14] by solving numerically some series equations. Let a be the circle radius and α, β, γ the angles in Fig. 3 determining the geometry of the problem (in Fig. 3 we have $\gamma \leq 0$). Then, the complex function

$$z = ln\left(\frac{Z - a\cos\gamma}{Z + a\cos\gamma}\right) + ln\sqrt{\frac{\cos((\alpha+\gamma)/2) \cdot \cos((\beta+\gamma)/2)}{\sin((\alpha-\gamma)/2) \cdot \sin((\beta-\gamma)/2)}} - i\left(\frac{\pi}{2} + \gamma\right) \tag{25}$$

where $ln(1) = 0$ and $Im(ln(Z_1)) \in (0, 2\pi)$, gives a conformal mapping of the domain in the Z-physical plane into the covered microstrip in the z-plane in Fig. 1 with particular parameters

$$h_1 = \pi, \; h_2 = \frac{\pi}{2} + \gamma \tag{26}$$

$$b = ln\sqrt{\frac{\sin((\beta-\gamma)/2) \cdot \cos((\alpha+\gamma)/2)}{\sin((\alpha-\gamma)/2) \cdot \cos((\beta+\gamma)/2)}}. \tag{27}$$

As the capacitance of a physical system is an invariant quantity by a conformal mapping, the linear capacitance of the structure in Fig. 3 can be determined by using the formulas given in Section II with geometrical parameters determined by relations (26), (27). We have also the relation

$$E_X - iE_Y = (E_x - iE_y) \cdot \frac{2a\cos\gamma}{Z^2 - a^2\cos^2\gamma}. \tag{28}$$

Thus, we can also determine the electric field intensity of the cylindrical line by means of the electrical field intensity of the equivalent planar microstrip structure.

VII. Other Cylindrical Lines Which Can Be Described by Means of Planar Microstrip Structures

We consider in this section other three cylindrical microstrip lines which can be described in quasi-TEM regime by means of some equivalent planar microstrip structures resulting from conformal mappings.

$$\tilde{\eta}(k) = \frac{2\exp(2k(h-h_1))/(1-\exp(-2kh_1)) + 2\epsilon_r \exp(2k(h-h_2))/(1-\exp(-2kh_e))}{\coth(kh_1) + \epsilon_r \coth(kh_e)} \tag{24}$$

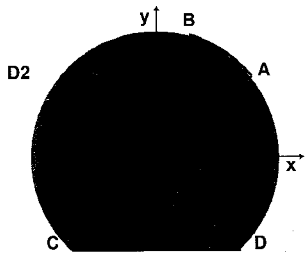

Fig. 4. Cylindrical structure A.

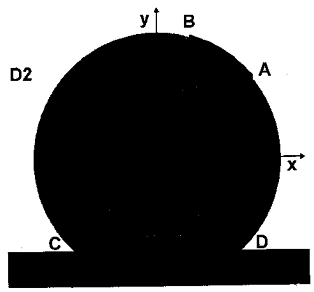

Fig. 5. Cylindrical structure B.

A. Structure A

The cylindrical microstrip line drawn in Fig. 4 consists of the perfectly conducting, infinitesimally thin, strips AB, CD placed on the circular surface and on the chord of a dielectric cylinder of circular segment cross section. By means of the complex variable function (25), where now we have $Im(ln(Z_1)) \in (-\pi, \pi)$ the domain in the Z-plane is mapped again in the domain in Fig. 1. The strip length $2b$ is determined by relation (27) and we have also

$$h_1 = \frac{\pi}{2} - \gamma, \; h_2 = \frac{3\pi}{2} + \gamma. \tag{29}$$

The capacity of the line, and hence the characteristic impedance, can be determined by using formulas given in Section II.

B. Structure B

The structure B, shown in Fig. 5, consists of a perfectly conducting strip AB (infinitesimally thin) on the surface of a dielectric cylinder with a circular segment cross section. The dielectric cylinder lies on a perfectly conducting grounded plane. The geometry of the problem can be characterized by means of the radius a and by angles α, β and γ. By the conformal mapping (25) the physical domain is mapped into the covered microstrip in Fig. 1 with the particular parameters

$$h_1 = \frac{\pi}{2} - \gamma, \; h_2 = \frac{\pi}{2} + \gamma. \tag{30}$$

The parameter b is again given by relation (27) and hence the capacitance of the line is given by relation (5).

C. Structure C

The cylindrical structure in Fig. 6 is composed of a perfectly conducting strip on the surface of a dielectric circular cylinder lying on a perfectly conducting grounded plane. The radius

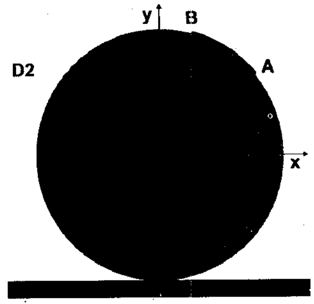

Fig. 6. Cylindrical structure C.

of the circular cross section is denoted by a and the position of the strip is described by angles α and β. By means of the conformal mapping

$$z = -\frac{1}{Z} + \frac{1}{4a}\left(\frac{\cos\alpha}{1+\sin\alpha} + \frac{\cos\beta}{1+\sin\beta}\right) - \frac{i}{2a} \tag{31}$$

the line in the Fig. 6 is conformally mapped into the line in Fig. 1 with the particular parameters

$$h_1 = \infty, \; h_2 = \frac{1}{2a} \tag{32}$$

and

$$b = \frac{1}{4a}\left(\frac{\cos\alpha}{1+\sin\alpha} - \frac{\cos\beta}{1+\sin\beta}\right). \tag{33}$$

Hence the capacitance of the line is given again by formula (5).

VIII. Numerical Examples

To compute the integrals entering in formulas (7)–(9) we have written

$$\int_0^\infty \frac{F(k, b)}{k}\, dk = \int_0^\infty f(t, b) \exp(-t)\, dt \tag{34}$$

where

$$f(t, b) = \frac{F(t/(ab), b)}{t} \exp(t). \tag{35}$$

The integral on the right-hand side of relation (34) can be computed by using a 32-point Gauss–Laguerre quadrature formula in double precision [15]. Numerical experiments indicated that if we put $a = 4$ for $b \geq 1$ and $a = 13$ for $b < 1$, we obtain at least five significant digits in the result for values of the parameter b in the range [0.005, 5.].

A. Application to a Symmetrically Covered Microstrip

To see how the given formulas work, we considered again the particular case of the symmetrical shielded microstrip ($h_1 = h_2 = h$). In this case a finite analytical expression for the line capacitance is available [16]

$$C_{exact} = 2(\epsilon_1 + \epsilon_2)\frac{K(k)}{K(k')} \tag{36}$$

where

$$k = \tanh\left(\frac{\pi b}{2h}\right), \; k' = \sqrt{1-k^2}. \tag{37}$$

Here $K(k)$ is the complete elliptical integral of the first kind.

TABLE I
COMPARISON OF THE C (EXACT) WITH C (APPR.) IN THE CASE OF SYMMETRICAL COVERED STRIPLINE

$h* = h/(2b)$	$C(exactly)$	$C1(apprx)$	$Rel.err.$
5.000	.969823E+00	.969826E+00	.26E-05
2.000	.134626E+01	.134627E+01	.37E-05
1.000	.187554E+01	.187555E+01	.50E-05
.500	.288224E+01	.288224E+01	-.21E-05
.250	.488254E+01	.488248E+01	-.12E-04
.125	.888254E+01	.888211E+01	-.49E-04
.100	.108825E+02	.108805E+02	-.19E-03
.075	.142160E+02	.142072E+02	-.61E-03

TABLE II
COMPARISON OF THE PROPOSED METHOD (PM), SPECTRAL DOMAIN METHOD (CE), AND SUBSTRIP METHOD (SS) IN THE CHARACTERISTIC IMPEDANCE CALCULATIONS

w/h	PM	CE	SS	PM	CE	SS
0.1	134.72	134.78	134.63	109.01	109.06	108.94
0.2	112.50	112.58	112.43	90.952	91.020	90.891
0.4	90.385	90.482	90.325	72.975	73.054	72.927
0.7	72.789	72.892	72.741	58.676	58.761	58.638
1.0	61.885	61.987	61.845	49.821	49.904	49.789
2.0	42.293	42.376	42.267	33.934	34.001	33.913
4.0	26.454	26.503	26.438	21.143	21.183	21.131
10.	12.726	12.745	12.717	10.125	10.140	10.118

TABLE III
COMPARISON OF THE PROPOSED METHOD (PM), SPECTRAL DOMAIN METHOD (CE), AND SUBSTRIP METHOD (SS) IN THE CHARACTERISTIC IMPEDANCE CALCULATIONS

w/h	PM	CE	SS	PM	CE	SS
0.1	94.670	94.718	94.605	65.578	65.612	65.534
0.2	78.955	79.015	78.902	54.658	54.699	54.622
0.4	63.312	63.381	63.270	43.787	43.835	43.759
0.7	50.870	50.943	50.837	35.143	35.194	35.120
1.0	43.166	43.238	43.139	29.792	29.843	29.773
2.0	29.357	29.415	29.338	20.212	20.249	20.200
4.0	18.258	18.282	18.247	12.536	12.555	12.528
10.	8.7260	8.7392	8.7197	5.9720	5.9808	5.9678

We compared the values for the capacitance given by proposed formulas (5)–(9) with the finite exact capacitance given by relation (36). The results are given in Table I. It is to be noticed that the results given by the new formulas are better than those obtained in [13]. In fact the maximum relative error is now 0.061% (for $h^* = h/(2b) = 0.075$) instead of 2% as was the corresponding value obtained in the cited paper.

B. Calculation of the Characteristic Impedance of an Open Microstrip Line

We used the formulas (5)–(10) for the evaluation of the characteristic impedance of the open microstrip line. The computed impedance values for the microstrip with different dielectric constants and $w/h = 2b/h_2$ ratios are shown in Table II and Table III. For purposes of comparison, the results for the same transmission line calculated by the spectral-domain method of Cheng and Everard [10] (denoted by CE), and those obtained by substrip method [2], [4], are also included in the tables (denoted by SS).

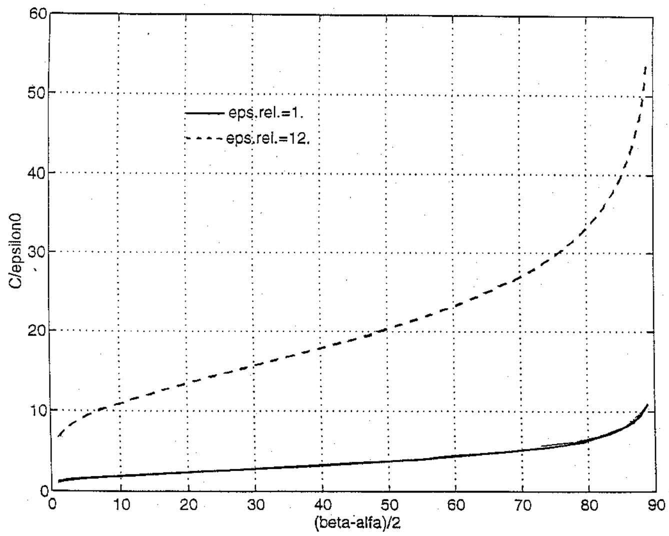

Fig. 7. Change of the capacitance with strip width.

It is easy to see that the proposed formulas (5)–(10) yields calculated impedance values which lies between the values obtained by CE and SS methods; the maximum relative error with respect to the values given by substrip method is less than 0.07% and the comparison with values computed by spectral domain approach gives a maximum difference less than 0.2%.

C. Application to the Cylindrical Microstrip Line Partially Embedded in a Perfectly Conducting Plane

To see how the methods works in the case of a cylindrical microstrip line resulting by a conformal mapping from a planar microstrip structure we considered the structure studied by Auda in [14]. The equivalent geometrical parameters h_1, h_2 and b were determined by formulas (26)–(27) where γ was set equal to zero. Further on, the capacitance of the line was computed by means of formulas (5)–(9). The change of the capacitance of a symmetrical cylindrical microstrip, partially embedded in a perfectly conducting plane, with the strip width is shown in Fig. 7. The change of the effective dielectric constant normalized capacitance $C(\epsilon_r)/C(\epsilon_r = 1))$ of the same cylindrical line is plotted in Fig. 8. For two differed wide strips and in the dielectric constant range $1 \leq \epsilon_r \leq 36$. It is to be noticed in the considered range that the linear dependence of the effective dielectric constant with respect to the dielectric constant for every strip width. In fact the slope of each of the effective dielectric lines depend upon the corresponding strip widths.

IX. CONCLUSION

The paper gives new formulas for computation of the capacitance and characteristic impedance of microstrip lines. The method requires only the numerical computation of integrals which can be evaluated by using the Gauss–Laguerre quadrature formulas. In the paper, it is shown how the methods apply in the case of covered microstrips on multilayer substrates and also in the case of anisotropic dielectrics. The method can also be applied to analyze some cylindrical microstrips. The

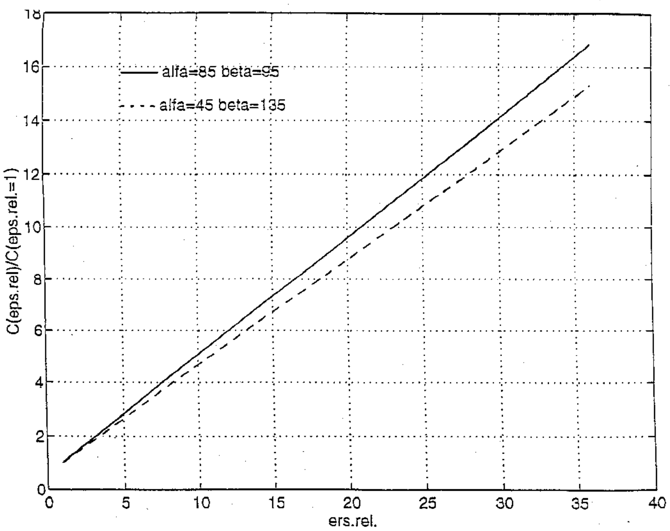

Fig. 8. Change of the capacitance with relative permittivity.

given numerical examples show that the method is easy to implement and highly accurate.

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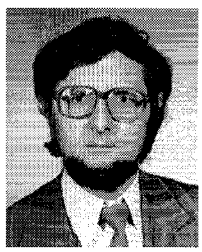

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